1883MNRAS..44...43A

The mean motions, again, are for Besselian mean days and Julian years, and to make them correct for the dates of our present *Nautical Almanac* they require to be multiplied by

$$\frac{\text{Sidereal Seconds in Leverrian Mean Day}}{\text{Sidereal Seconds in Besselian Mean Day}}.$$

Hansen's or other Tables of the Moon will require a corresponding set of corrections to adapt them to our Observatory Times.

It is, however, unnecessary to correct the Tables. All we need do is to recollect that the data in our present *Nautical Almanac* are the direct result of the Tables, and that they must be corrected for the difference between the Mean Time used in making the Tables and the Leverrian Mean Time of the Observatories. Thus, if the difference between the Mean Time which Hansen's Tables are referred to and Leverrian Time is n^s on any day, the latter being the smaller we must compare the Moon's Right Ascension at Transit with that which according to the almanac she would have had n^s after the Transit.

When the data of the Tables have been entirely determined from observations reduced as I have described, then the Tables require only this correction of date. But there have been used some isolated old observations such as eclipses of the Sun in the Lunar Tables and Transits of *Venus* and *Mercury* whose times can only be determined by solar observations, or are given by observers in Apparent Time without means of referring to stars. In these cases the Mean Times are Leverrian, or *True* Mean Times, within the limits of observation, and we should expect a discordance with the calculations made from the provisional data. By using them to correct the data of the Tables on the hypothesis that these times were Besselian, error has been introduced. A marked case is that of the Secular Acceleration of the Moon's Mean Motion which has been determined practically from very old eclipses compared with the Tables whose mean motion came from modern observations.

1883, *Oct.* 31.

Remarks on Mr. Stone's Explanation of the Large and Increasing Errors of Hansen's Lunar Tables by means of a supposed Change in the Unit of Mean Solar Time. By Prof. J. C. Adams, M.A., F.R.S.

In some recent communications to the Royal Astronomical Society Mr. Stone contends that the mean solar day in use before 1864—when Le Verrier's Solar Tables were substituted for Bessel's in calculating the sidereal time at mean noon given in the *Nautical Almanac*—differs from the mean solar day adopted since that time.

1883MNRAS..44...43A

In the *Monthly Notices*, vol. xliii. p. 403, Mr. Stone states that the consequent error in our present reckoning in time is increasing at about the rate of $1^s{\cdot}46$ per annum, and in the same volume, p. 335, he adduces this supposed error in explanation of the increasing errors of Hansen's Lunar Tables.

That this view of Mr. Stone's is erroneous may, I think, be shown by very simple considerations.

The only mean Sun known to astronomers is an imaginary body which moves uniformly in the equator at such a rate that the difference between its Right Ascension and that of the true Sun consists wholly of periodic quantities.

These periodic terms are due to the obliquity of the ecliptic, the eccentricity of the Earth's orbit, and also to the small perturbations of the Earth's motion about the Sun.

The difference between the Right Ascensions of the two bodies at any moment is called the Equation of Time.

The instant of Mean Noon is determined by the transit of this imaginary Mean Sun over the meridian of a given place just as the instant of Apparent Noon is determined by the transit of the true Sun over the same meridian.

Hence, the mean time, according to the definition of it above given, may be determined by observation of the transit of the true Sun over the meridian, subject only to the small error to which all transit observations are liable, and also to the extremely small error which is possible in the theoretical expression for the equation of time. When this mode of determining the mean time is employed, no accumulation of error in proportion to the interval of time from a given epoch is possible.

If, as it is frequently convenient to do, we wish to determine the mean solar time by means of the sidereal time supposed to be known, without having to make a transit observation of the Sun, we must employ the sidereal time at mean noon calculated from the proper formula or from the Solar Tables. This sidereal time at mean noon is equal to the Sun's mean longitude at mean noon corrected by the equation of the equinoxes in Right Ascension.

In order to find the mean time correctly in this way it is necessary to employ the correct value of the Sun's mean longitude, and any error in the assumed value of this quantity will produce an equivalent error in the mean time deduced.

Any such error can be at once checked and corrected by observation of the Sun's transit over the meridian.

If we wilfully refuse to check our results by solar observations, the error in the determination of the mean time by means of the sidereal time would, no doubt, increase in proportion to the interval of time from a certain epoch. Practically, however, it would be intolerable to use Solar Tables which were grossly erroneous, and long before the error of time became important the tables would be replaced by more accurate ones.

For many years previously to 1864 Bessel's formula had been

1883MNRAS..44...43A

employed in the *Nautical Almanac* for the calculation of the sidereal time at Greenwich mean noon.

In 1864 the error of Bessel's formula amounted to rather more than half a second of time, and accordingly in that and subsequent years the sidereal time at mean noon was deduced from Le Verrier's Solar Tables, which gave much more accurate results.

Now it is contended by Mr. Stone that by the change thus introduced into the *Nautical Almanac* the unit of mean solar time was practically altered to such a degree that at the end of 1881 the difference in the count of mean solar time amounted to nearly 27 seconds, and that the difference is increasing at the rate of about 1·46 seconds per annum.

It is clear, therefore, that if no such change had been made in the *Nautical Almanac*—that is, if Bessel's formula had continued to be employed—no such change of the unit of time would have taken place.

Let us see then, what difference this would have made in the count of mean solar time as derived from sidereal time when compared with the count found by means of our present *Nautical Almanac.*

Bessel's formula for the sidereal time at Greenwich mean noon of Jan. 1 in any year is given in the prefaces to the *Nautical Almanacs* from 1834 to 1863 inclusive. In 1864 and subsequent years the sidereal time at Greenwich mean noon is derived from Le Verrier's tables.

The following little table shows the sidereal time at Greenwich mean noon of Jan. 1 as calculated for every fifth year from 1860 to 1885 by Bessel's formula, and as taken from the several *Nautical Almanacs* :—

	By Bessel's Formula.			From *Nautical Almanac.*				Diff.
	h	m	s	h	m	s		s
1860	18	41	28·87	18	41	28·87	Bessel's formulæ employed	0·00
1865	18	44	35·36	18	44	35·92	Le Verrier's Tables employed	0·56
1870	18	43	43·87	18	43	44·44	,, ,,	0·57
1875	18	42	54·47	18	42	55·06	,, ,,	0·59
1880	18	42	5·95	18	42	6·56	,, ,,	0·61
1885	18	45	11·73	18	45	12·37	,, ,,	0·64

Hence we see that the difference of sidereal times at mean noon in consequence of the change from Bessel's formula to Le Verrier's Tables, which amounted to 0^{s}·56 in 1865, had increased to 0^{s}·64 in 1885. That is, the difference increases at the rate of 0^{s}·08 in twenty years, or of 0^{s}·02 in five years.

But according to Mr. Stone's theory as shown in his tabular comparisons of mean solar times computed from sidereal times by means of the *Nautical Almanac* and of those sidereal times

1883MNRAS..44...43A

"corrected to agree with Bessel's sidereal times," the differences would be as follows:—

	s		s
1865	2·0	1875	16·6
1870	9·3	1880	23·9

and at the end of 1881 the difference would have increased to $26^s{\cdot}8$; so that the increase in five years would be $7^s{\cdot}3$ instead of $0^s{\cdot}02$ as above. In fact the difference according to Mr. Stone's theory is just 365 times as great as it should be.

The origin of this enormous discrepancy between Mr. Stone's theory and the fact is readily seen by considering that mean solar time is measured, not by the Sun's mean motion in *longitude*, as Mr. Stone's theory supposes, but by the motion of the mean Sun in *hour angle*, which is about 365 times greater in amount. Hence any small error in the determination of the Sun's mean motion in longitude causes a proportionate error of only about a 365th part of the amount in the interval of mean solar time as inferred from the interval of sidereal time. In fact, if n denote the Sun's mean motion in longitude in a mean solar day, then the length of the mean solar day will be to the sidereal day in the ratio of

$$360^\circ + n : 360^\circ.$$

If now $n + dn$ denote another slightly different determination of the Sun's mean motion in longitude in a mean solar day, the ratio of the length of a mean solar to that of a sidereal day will become

$$360^\circ + n + dn : 360^\circ.$$

Hence the measure of a mean solar day when expressed in sidereal time will be increased in the ratio of

$$360^\circ + n + dn : 360^\circ + n,$$

or

$$1 + \frac{dn}{360^\circ + n} : 1.$$

Since 360° is nearly 365 times n, this ratio will be

$$1 + \frac{1}{366}\ \frac{dn}{n} : 1 \text{ nearly},$$

Whereas, according to Mr. Stone's theory, this ratio should be

$$1 + \frac{dn}{n} : 1.$$

It has been already remarked that it is convenient practically to determine the mean solar time from the sidereal time, but in order to do this correctly, it is of course necessary

1883MNRAS..44...43A

to employ the correct value of the Sun's mean longitude. At the present time Bessel's value of the Sun's mean longitude is about $0^{s}{\cdot}6$ in error, and therefore the mean solar time inferred by means of it from the sidereal time would be in error to the same amount. The mean longitude found from Le Verrier's Tables is much nearer to the truth, and therefore the mean solar time found from the sidereal time by using this value would be much more nearly correct.

It must not be forgotten however that, as we have already stated, the mean solar time may be derived from observations of the transit of the Sun over the meridian, without employing the sidereal time at all. Apparent solar time, which is found directly from observation of the Sun is converted into *mean* solar time by applying the equation of time, which is known from the solar theory, without reference to the sidereal time.

On the Change in the Unit of Time implied in the Substitution of Hansen's for Bessel's Expression for the Longitude of the mean Sun. By Prof. A. Cayley, M.A., F.R.S.

I have before me Mr. Stone's paper "An Explanation of the Principal Cause of the Large Errors &c.", *Monthly Notices*, vol. xliii. (1883) pp. 339–345, in which Mr. Stone arrives at the conclusion that the effect of the substitution of Hansen's expression for the longitude of the mean Sun in place of that of Bessel is to change the unit of time in the ratio of $1+x$ to 1, where

$$x = -\frac{0{\cdot}055871}{1296027{\cdot}674055}.$$

Prof. Adams considers, I think rightly, that the true value of x is about $\frac{1}{365}$ of this value.

I wish in the present paper to present the question in what appears to me to be the most distinct manner.

Bessel's expression for the longitude of the mean Sun, 1850 Jan. 1, Paris mean noon $+\,t$ is

$$l = 280^{\circ}\ 46'\ 36''{\cdot}12 + 1296027''{\cdot}618184\,.\,t + 0''{\cdot}0001221805\,.\,t^2,$$

say $\quad = \quad$,, $\quad + \mathrm{L}t \quad$,,

the unit of time being *supposed* to be a Julian year of 365·25 mean solar days, and only the term in t being here attended to.

I infer that according to Bessel the Sun's mean hour-angle from the meridian of Paris would be

$$h, = 365{\cdot}25 \times 360^{\circ}\,.\,t,$$

viz. t increasing by $\dfrac{1}{365{\cdot}25}$ (=one mean solar day) h would increase by 360°. Say this value is $h = \mathrm{H}t$.